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Natural Convection in a Tilted Square Enclosure Having Heat Generating Solid Body and with Various Thermal Boundaries

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Abstract

Steady laminar natural convection around a heat conducting and generating solid body inside a two-dimensional tilted square enclosure with different thermal boundaries are investigated numerically. The mathematical model is governed by the coupled equation of mass, momentum and energy. These equations are solved using SIMPLE algorithm by means of the finite-volume method together with under-relaxation technique. Effect of angle of inclination of enclosure, aspect ratio of solid-enclosure, temperature difference ratio of solid-fluid and the thermal conductivity ratio of solid-fluid are analyzed. The fluid circulation and the corresponding heat transport inside the enclosure are represented in terms of streamlines and isotherms respectively.

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Keywords: Natural convection; Thermal conductivity ratio; SIMPLE algorithm; Finite volume method.

1. Introduction

Natural convection heat transfer induced in enclosures has received considerable attention because of its numerous applications in geophysics and energy related engineering problems. The natural convection which is also known as buoyancy-driven flows occurs in many practical problems. Some of the commonly used buoyancy driven flows are observed in nature; such as atmospheric fronts, katabolic winds etc., and in industry such as boilers, nuclear reactor systems, energy storage and conservation, fire control, chemical food and metallurgical industries. The first work on the numerical simulation of natural convection heat transfer inside enclosure was done by Davis [1] which was standard benchmark problem of buoyancy-driven flow in an enclosed domain.

Effect of existence of obstacles within an enclosure was one of the interesting investigations for researchers. House et al. [2] and Oh et al. [3] analyzed the steady natural convection in an enclosure with a heat generating conducting body numerically. They studied the effect of Reyleigh number and temperature difference across an enclosure on the fluid flow and heat transfer. Roychowdhury et al. [4] surveyed the natural convective flow and heat transfer features for a heated body placed in a square enclosure with various thermal boundary conditions. Khozeymehnezhad and Mirbozorigi [5] compared the natural convection around a circular cylinder with a square cylinder inside a square

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Nomenclature

A	Amplitude
g	Acceleration due to Gravity(ms^{-2})
K	Thermal Conductivity($Wm^{-1}K^{-1}$)
$\dot{q}$	Heat Generation Per Unit Volume
T	Temperature(K)
u, v	Dimensional Velocity Components
x, y	Dimensional Co-ordinates
α	Thermal Diffusivity(m^2s^{-1})
β	Coefficient of Thermal Expansion(K^{-1})
θ	Non-dimensional Temperature
μ	Dynamic Viscosity($Kg.s.m^{-1}$)
ν	Kinematic Viscosity(m^2s^{-1})
ϕ	Angle of Inclination
χ	General Variable
ρ	Density(kgm^{-3})
c	cold
f	fluid
h	hot
s	solid

enclosure. Singh et al. [6] analyzed the heat transfer and fluid flow due to natural convection in air around heated square cylinder of different sizes inside an enclosure. Conjugate natural convection inside the inclined square enclosure with a conducting block was investigated out by Das and Readdy [7].

Buoyancy driven flow with various thermal boundary conditions (isoflux, isothermal, linearly heated, sinusoidal heating and adiabatic) has many industrial applications such as energy efficient design of building and room etc., Sathiyamoorthy et al. [8] investigated the effect of different thermal boundary conditions on natural convection flows with a square enclosure. They found that non-uniform heating of the bottom wall enhances the heat transfer rate than the uniform heating case for all Rayleigh number. At the same time, the effect of different thermal boundary conditions at bottom wall on natural convection in cavities was studied by Aswatha et al. [9].

Moreover, Effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cuwater nanofluid using CVFEM was studied by Sheikholeslami et al. [10]. Prasopchingchana et al. [11] analyzed the natural convection of air in an inclined square enclosure. They reported that the angles of the inclination of the enclosure giving the maximum average Nusselt numbers are $\phi = 110^o$ for $Ra = 10^3$ and $\phi = 130^o$ for $3X10^3 < Ra = 1X10^4$. Natural convection in an inclined square enclosure containing internal energy sources are surveyed by Islam et al. [12] and they concluded that average Nusselt number was decreased with an increment of the tilted angle. In general, optimum heat transfer performance was obtained at zero inclination angle.

Present work revels the numerical investigation of natural convection in a tilted square enclosure having heat generating solid body and with different thermal boundary conditions. The heat transfer results explain the importance of the non-dimensional parameters like angle of inclination, aspect ratio, temperature difference ratio and thermal conductivity in the natural convection regime.

2. Mathematical Formulation

The present flow is considered steady, laminar, incompressible and two-dimensional. Figure 1 shows the physical configuration of the present problem. The enclosure dimensions are defined by L for width and H ($H = L$) for height. A heat generating solid square body of width W is placed at its centre. The enclosure is isothermally heated from the bottom horizontal wall with a uniform constant temperature T_h and the right vertical wall with temperature T_c

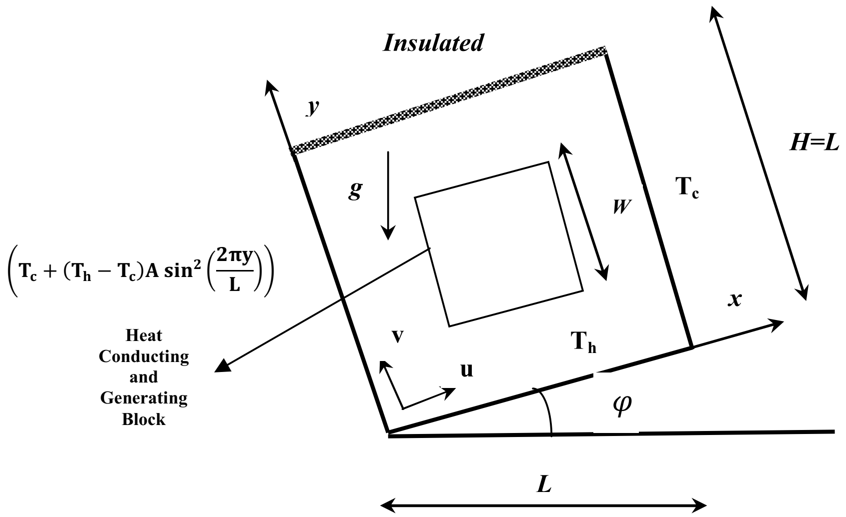

Fig. 1. Schematic of the Problem.

($T_h > T_c$). The left vertical wall is heated with sinusoidal ($T_c + (T_h - T_c)Asin^2(2\pi y/L)$) while the remaining top wall is considered as perfectly insulated. Fluid having constant properties is assumed where buoyancy effect is included through Boussinesq approximation. Under this assumption, equations of continuity, momentum and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u + g\beta(T - T_c)sin\phi \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v + g\beta(T - T_c)cos\phi \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T. \tag{4}$$

Energy equation for the solid body is,

$$K_s\frac{\partial^2 T_s}{\partial x^2} + K_s\frac{\partial^2 T_s}{\partial y^2} + \dot{q} = 0. \tag{5}$$

Dimensional boundary conditions are:

$$x = 0: \ 0 \le y \le L, \ u = v = 0, \ T = T_c + (T_h - T_c)A\sin^2(\frac{2\pi y}{L}) \tag{6}$$

$$x = L: \ 0 \le y \le L, \ u = v = 0, \ T = T_c \tag{7}$$

$$y = 0: \ 0 \le x \le L, \ u = v = 0, \ T = T_h \tag{8}$$

$$y = L: \ 0 \le x \le L, \ u = v = 0, \ \frac{\partial T}{\partial y} = 0. \tag{9}$$

Boundary condition at the fluid-solid interface:

$$K_f\left(\frac{\partial T}{\partial n}\right)_{fluid} = K_s\left(\frac{\partial T_s}{\partial n}\right)_{solid} \tag{10}$$

where n is the vector acting normal to the boundary. To make the above equations dimensionless, the following non-dimensional variables and parameters are introduced.

$$(X,Y) = \frac{(x,y)}{L}, \quad (U,V) = \frac{(u,v)}{\alpha/L}, \quad P = \frac{pL^2}{\rho\alpha^2}, \quad \theta = \frac{T-T_c}{T_h-T_c}, \quad \theta_s = \frac{T_s-T_c}{T_h-T_c}, \quad (T_h > T_c).$$

$Ra = \dfrac{g\beta(T_h - T_c)L^3}{\alpha\nu}$ Rayleigh number, $Pr = \dfrac{\nu}{\alpha}$ Prandtl number, $K^* = \dfrac{K_s}{K_f}$ solid fluid thermal conductivity ratio, $\Delta T^* = \dfrac{(\frac{\dot{q}W^2}{K_f})}{(T_h - T_c)}$ temperature difference ratio and $A^* = \dfrac{W^2}{L^2}$ aspect ratio.

A very important property of the natural convection flows is to calculate the rate of heat transfer along the walls of the enclosure. The dimensionless parameter named Nusselt number stands for this quantity. The average Nusselt number can be calculated as

$$\overline{Nu}_{left\,wall} = \overline{Nu}_{right\,wall} = \int_0^L -\left(\frac{\partial T}{\partial x}\right)_{wall} dy.$$

$$\overline{Nu}_{bottom\,wall} = \int_0^L -\left(\frac{\partial T}{\partial y}\right)_{y=0} dx.$$

3. Numerical Procedure

The governing equations for the simulations of fluid flow inside the enclosure are numerically solved according to their boundary conditions. The results are obtained with finite volume method based FORTRAN 77 code. The SIMPLE algorithm by Patanker [13] is adopted to solve the velocity-pressure coupling. The power-law scheme is employed to discretize the convective and diffusive terms of the governing equations. The resulting sets of algebraic equations are solved by the tri-diagonal matrix algorithm (TDMA). The convergence criterion is applied for all variables which is imposed as $\left|\dfrac{\chi_{i,j}^{n+1} - \chi_{i,j}^{n}}{\chi_{i,j}^{n+1}}\right| < 10^{-6}$.

Table 1. Comparison table for Nusselt number.

Ra	Solid Body	Davis [1]	House et al.[2]	Oh et al.[3]	Present
10^3	No	1.118	–	1.119	1.117
10^3	No	4.519	–	4.565	4.550
10^5	Yes($A^* = 0.25, K^* = 0.2$)	–	4.624	4.626	4.651
10^5	Yes($A^* = 0.25, K^* = 5.0$)	–	4.325	4.327	4.457

The problem of buoyancy-driven enclosure is used as a suitable benchmark for testing the natural convection codes in the literature. The present results are compared with the similar results of Davis [1], House et al. [2] and Oh et al. [3]. Table1 shows the comparison values of Nusselt number for the case of enclosure with or without solid body at its centre for various Reyleigh number and thermal conductivity ratio. A good agreement was observed between the previous works and the present one. Based on this successful validation, this code can be used to compute the present problem. A staggered grid system was employed for the grid independency test and it is decided to utilize the 141X141 grid size for the final calculation. All calculations are carried out on Pentium N3530 quad core personal computer in 2,73,114 seconds.

4. Result and Discussion

In this work, a numerical simulation of the influence of boundary conditions on natural convection in a tilted square enclosure with heat generating solid body has been performed. The study focuses on effects of the angle of inclination,

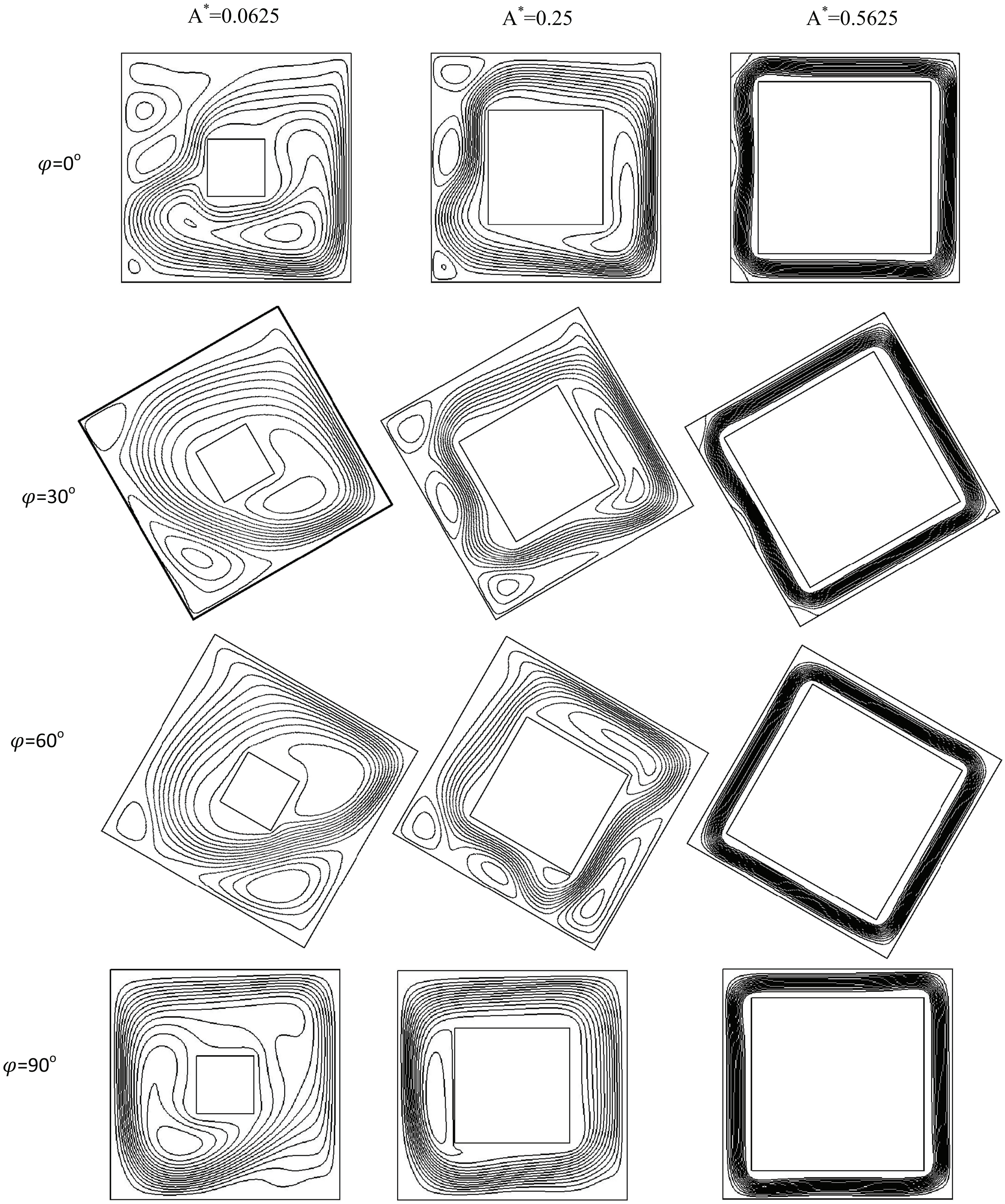

Fig. 2. Streamlines for fixed $Ra = 10^6$, $K^* = 2$,$\Delta T^* = 4$ and for different $A^*(0.0625, 0.25, 0.5625)$, $\phi(0^o, 30^o, 60^o, 90^o)$.

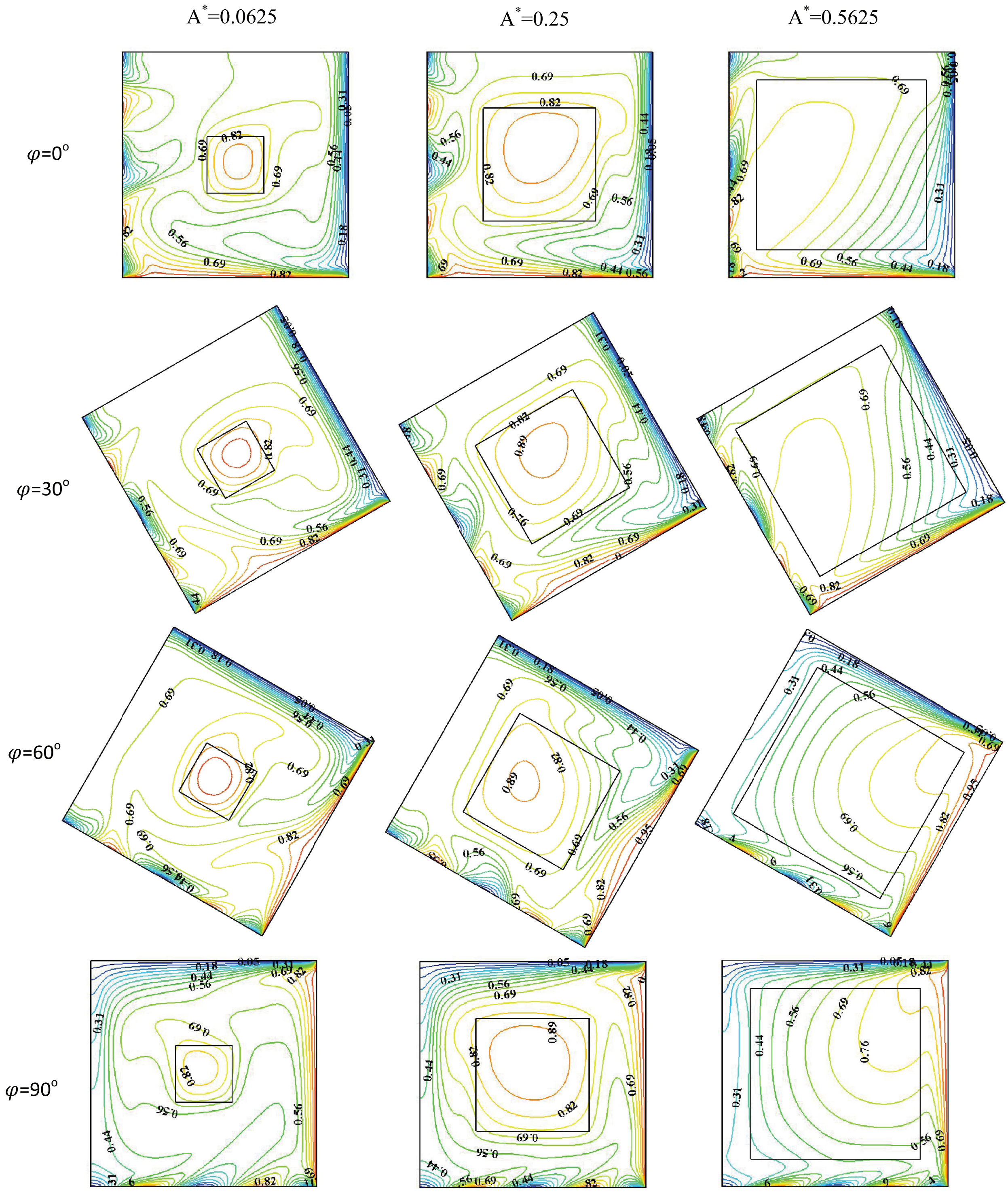

Fig. 3. Isotherms for fixed $Ra = 10^6$, $K^* = 2$,$\Delta T^* = 4$ and for different $A^*(0.0625, 0.25, 0.5625)$, $\phi(0^o, 30^o, 60^o, 90^o)$.

aspect ratio of the solid body and enclosure, temperature difference ratio of fluid and solid, thermal conductivity ratio of fluid and solid on the flow and temperature fields. Which are ranging from 0^o to 90^o, 0.0625 to 0.5625, 0 to 50 and 1 to 5 respectively. The Rayleigh number is fixed as 10^6 and the Prandtl number as 0.71.

For the low value of the heat generation inside the enclosure ($\Delta T^* = 4$) and for the high value of Rayleigh number ($Ra = 10^6$), the buoyancy force accelerates the flow and thus convection current dominates the heat transfer. This is only for small A^*. From figures 2 and 3, it is seen that for small A^* fluid circulates freely inside the enclosure. The fluid flow inside the enclosure is high around the solid body and low at the walls of the enclosure owing to no slip boundary conditions. Large contour induced by the buoyancy force occupies the major part of the enclosure and small contours caused by sinusoidal heating appear near the left wall. Heat transfer is more efficient for the case of $A^* = 0.0625$ and $A^* = 0.25$. As A^* increases, volume inside the enclosure decrease, the flow intensity is not much

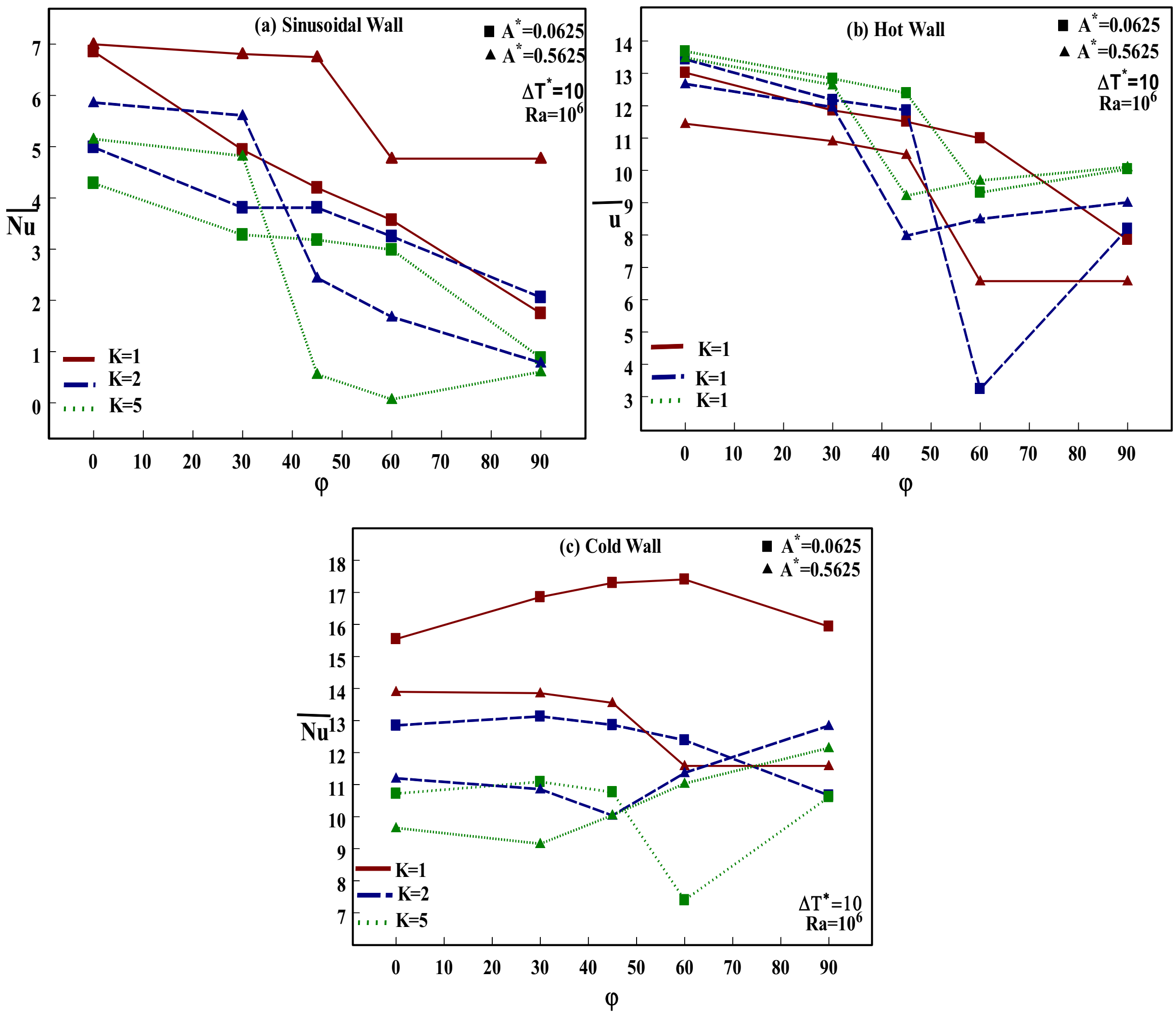

Fig. 4. Effect of various parameters on Nusselt number for different ϕ and thermal boundaries.

better and an inadequate flow pattern is observed. As the angle of inclination is zero heat transfer is high inside the enclosure but for the increase in inclination angle leads to the decrease in heat transfer all over the enclosure.

Figure 4. shows the average Nusselt number graph against various angle of inclination of the enclosure. Here $\Delta T^* = 10$, $Ra = 10^6$ are fixed and A^* (0.0625 and 0.5625) and K^*(1, 2 and 5) are varied. As the angle of inclination increases, the average Nusselt numbers get decreased along all the walls of the enclosure. Therefore, the increment of the tilted angle suppresses the heat transfer all over the enclosure. When thermal conductivity ratio (K^*) of the solid and fluid region is 1, it indicates both (for solid and fluid) thermal conductivity are equal. Hence only convection mode of heat transfer takes place. As this ratio increases, conduction mode begins to takes place. Thus the convection current declaims for increasing K^*. Maximum heat transfer takes place in the cold wall for all angle of inclination and for $A^* = 0.0625$, $K^* = 1$. And the minimum is reached for the case of $A^* = 0.5625$, $K^* = 5$ and $\phi = 60^o$.

Conclusion

- For the low value of the ΔT^* inside the enclosure ($\Delta T^* = 4$) and for the high value of Ra, the buoyancy force accelerates the flow and thus convection current dominates the heat transfer. This is only for small A^*.
- Increment in ϕ suppresses the heat transfer all over the enclosure also the convection current declaims for the increasing K^*.
- Heat transfer attains its maximum in the cold wall for all ϕ and for $A^* = 0.0625$, $K^* = 1$. And the minimum is reached for the case of $A^* = 0.5625$, $K^* = 5$ and $\phi = 60^o$.

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References

[1] G.D.V. Davis, Natural convection of air in a square cavity: A benchmark numerical solutions, Int. J. of Num. Meth. in Fluids. 3 (1983) 249-264.
[2] J.M. House, C. Beckermann, T.F. Smith, Effect of centered conducting body on natural convection heat transfer in an enclosure, J. Num. Heat Trans. Part A. 18 (1990) 213-225.
[3] J.Y. Oh, M.Y. Ha , K.C. Kim, Numerical study of heat transfer and flow of natural convection in an enclosure with a heat-generating conducting body, J. Num. Heat Trans. Part A. 31 (1997) 289-303.
[4] D.G. Roychowdhury, S.K. Das, T.S. Sundararajan, Numerical simulation of natural convection heat transfer and fluid flow around a heated body inside an enclosure, J. Heat and Mass Trans. 38 (2002) 565576.
[5] H. Khozeymehnezhad, S.A. Mirbozorigi, Comparison of natural convection around a circular cylinder with a square cylinder inside a square enclosure, J. Mech. Eng. and Auto, 2 (2012) 176-183.
[6] R.K. Singh, K.B. Sahu, T.D. Mishra, Analysis of heat transfer and flow due to natural convection in air around heated square cylinder of different sizes insides an enclosure, Int. J. Eng. Res. and Appl. 3 (2013) 766-771.
[7] M.K. Das, K.S.K. Reddy, Conjugate natural convection heat transfer in an inclined square cavity containing a conducting block, Int. J. Heat and Mass Trans. 49 (2004) 49875000.
[8] M. Sathiyamoorthy, T. Basak, S. Roy, I. Pop, Steady natural convection flows in a square cavity with linearly heated side wall(s), Int. J. Heat and Mass Trans. 50 (2007) 766-775.
[9] Aswatha, C.J.G. Gowda, S.N. Sridhara, K.N. Seetharamu, Effect of different thermal boundary conditions at bottom wall on natural convection in cavities, J. Eng. Sci. and Tech. 6 (2011) 109 - 130.
[10] M.Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, S. Soleimani, Effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with *Cu* water nanofluid using CVFEM, Int. J. Adv. Pow. Tech. (2013) Article in Press.
[11] U. Prasopchingchana, W. Pirompugd, P. Laipradit, K. Boonlong, Numerical study of natural convection of air in an inclined square enclosure, Int. J. Materials. 1 (2013) 131-135.
[12] M.T. Islam, S. Saha, M.A.H. Mamun, G. Saha, Natural convection in an inclined square enclosure containing internal energy sources, J. Mech. Eng., Tran. Mech. Eng. Div. ME37 (2007) 24-32.
[13] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Mc Graw Hill, Washington DC, (1980).